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General Certificate of Education (A-level)
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Mathematics MM2B

(Specification 6360)

Mechanics 2B

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM2B

Q	Solution	Marks	Total	Comments
1(a)	KE = $\frac{1}{2}\times 58\times 2^2$	M1		M1: Correct fully substituted expression for KE.
	= 116 J	A1	2	A1: CAO
(b)	Change in PE: $mgh = 58 \times 9.8 \times 7$	M1		M1: Expression for PE with 58 and 9.8 or 9.81 with 6 or 7 for the height (or 11 and 4, 11 and 5 or 10 and 4).
	= 3978.8	A1		A1: Accept 3980 or 3970 or 3978 or 3979 or 3978.8. Accept 3982 or 3983 or 3980.
	KE = 3978.8 + 116 J = 4094.8 J	M1		M1: Adding their two previous answers.
	Speed of Kim is $\sqrt{\frac{4094.8}{\frac{1}{2}\times 58}}$	dM1		dM1: Seeing expression for v (not v^2), dependent on second M1
	= 11.88 m s^{-1} = 11.9 m s^{-1}	A1	5	A1: Accept 11.88 or 11.8 or 11.9 Accept 11.88 or 11.8 or 11.9 or AWRT 11.89 from $g = 9.81$.
				Obtaining $v=\sqrt{u^2+2gh}$ followed by incorrect substitution M0M1M1, unless h is 6 or 7, which is M1M1M1
				11.0 (from $h = 6$) M1M1M1
				$v=\sqrt{2^2+2\times g\times 7}$ M1M1M1 $=\sqrt{141.2}$ A1 $=11.9$ A1
				$v=\sqrt{4+14g}$ M1M1M1A1 $=11.9$ A1
				$v=\sqrt{2^2+12g}$ M1M1M1
	Total		**7**	

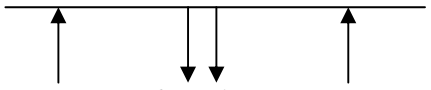
MM2B (cont)

Q	Solution	Marks	Total	Comments
2	$\bar{X} = \dfrac{2\times 9+3\times 2+8\times 3+7\times 6}{2+3+8+7}$	M1		M1: Expression for $\bar{X}$ with no more than one error in the numerator and correct denominator.
	$= \dfrac{90}{20}$ or 4.5	A1		A1: Correct distance. Accept $\dfrac{9}{2}$ or $\dfrac{90}{20}$ or equivalent.
	$\bar{Y} = \dfrac{2\times 6+3\times 4+8\times 8+7\times 11}{20}$	M1		M1: Expression for $\bar{Y}$ with no more than one error in the numerator and correct denominator.
	$= \dfrac{165}{20}$ or 8.25	A1		A1: Correct distance. Accept $\dfrac{33}{4}$ or $\dfrac{165}{20}$ or equivalent
	$\therefore$ Centre of mass is at (4.5, 8.25)	A1F	5	A1: Correct coordinates; dependent on M1 M1 Do not accept $\dfrac{90}{20}$ etc at this stage. SC4: For final answer (8.25, 4.5) award 4 marks. Moments about *B*, (2.5, 4.25) SC2
	Total		**5**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\mathbf{a} = \frac{\mathrm{d}v}{\mathrm{d}t}$			
	$\mathbf{a} = -8\mathrm{e}^{-2t}\mathbf{i} + (6-6t)\mathbf{j}$	M1 A1 A1	3	M1: Differentiating with either of the two components correct. Do not need to see **i** or **j**. A1: Correct **i** component. A1: Correct **j** component.
(b)(i)	Using $\mathbf{F} = m\mathbf{a}$			
	$\mathbf{F} = 5\times\{-8\mathrm{e}^{-2t}\mathbf{i} + (6-6t)\mathbf{j}\}$	M1 A1	2	M1: Multiplying their acceleration by 5, even if not a vector. A1: Correct expression.
	$= -40\mathrm{e}^{-2t}\mathbf{i} + (30-30t)\mathbf{j}$			
(ii)	Magnitude of $\mathbf{F}$ is			
	$\{(-40)^2 + (30)^2\}^{\frac{1}{2}}$	M1		M1: Finding magnitude from two non-zero terms. Must add terms and square root. Condone $\{(40)^2 + (30)^2\}^{\frac{1}{2}}$
	$= 50$	A1	2	A1: Correct answer only. In this part, condone lack of negative signs in expression for force in (b) (i).
(c)	When **F** acts due west, **j** component is zero	M1		M1: Putting **j** component equal to zero.
	$30 - 30t = 0$ $t = 1$	A1	2	A1: Correct time.
(d)	$\mathbf{r} = -2\mathrm{e}^{-2t}\mathbf{i} + (3t^2 - t^3)\mathbf{j} + \mathbf{c}$	M1 A1 A1		M1: Integration with either of the two components correct. Do not need to see **i** or **j**. A1: Correct **i** component. A1: Correct **j** component. Condone lack of + **c**
	When $t = 0$, $\mathbf{r} = 6\mathbf{i} + 5\mathbf{j}$ $\therefore$ $\mathbf{c} = 8\mathbf{i} + 5\mathbf{j}$	dM1		dM1: Finding **c** using $6\mathbf{i} + 5\mathbf{j}$ and $\mathrm{e}^0 = 1$.
	$\therefore$ $\mathbf{r} = (8 - 2\mathrm{e}^{-2t})\mathbf{i} + (5 + 3t^2 - t^3)\mathbf{j}$	A1	5	A1: Correct position vector.
	Total		**14**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	 R_C or C or $38g$; $65g$ Or 637 ; $17g$ Or 166.6 ; R_D or D or $44g$	B1 B1	2	B1: Two weights correct and in correct relative positions. B1: Two upward reaction forces, labelled differently. Note all forces must be shown as arrows and have labels. Condone use of $g = 9.81$ for calculating weights.
(b)	Taking moments about C $3 \times 17g + 2.6 \times 65g = 44g \times d$	B1 M1 A1		B1: Seeing 2.6. M1: Three term moment equation including $17g$, $65g$ and $44g$ or 17, 65 and 44, with different distances for the $17g$ and $65g$. A1: Correct equation.
	$44d = 220$ $d = 5$			
	Distance is $5 - 4.6 = 0.4$ m	A1	4	A1: Correct final answer.
	Alternative $R_C = 38g$ Taking moments about D $38g(4.6 + x) = 65g(2 + x) + 17g(1.6 + x)$ $174.8 - 130 - 27.2 = 44x$ $x = 0.4$	(B1) (M1) (A1) (A1)		Could take moments about any other point
(c)	Gravitational force (centre of mass or weight) at mid-point (or centre) of the plank	E1	1	E1: Correct explanation.
	Total		**7**	
5(a)	$90\,\text{km h}^{-1} = 90 \times \frac{1000}{3600}\,\text{m s}^{-1}$ $= 25\ \text{m s}^{-1}$ **AG**	B1	1	B1: Must see $\frac{1000}{3600}$ or $\frac{1000}{60^2}$.
(b)	Resistance is 5000 N Using power = force × velocity = 5000 × 25 = 125 kW	B1 M1 A1	3	B1: Obtaining 5000. M1: Using $P = Fv$ with 25 and their F. A1: Correct final answer, must be in kW. 125W or 125 000 W B1M1 125 B1M1A1
	Total		**4**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	Using $F = ma$ $-2mv^{\frac{5}{4}} = m\frac{dv}{dt}$ $\therefore \frac{dv}{dt} = -2v^{\frac{5}{4}}$ **AG**	B1	1	B1: Must see $-2mv^{\frac{5}{4}} = m\frac{dv}{dt}$ or $-2mv^{\frac{5}{4}} = ma$ and correct final answer.
(b)	$\int \frac{dv}{v^{\frac{5}{4}}} = -2\int dt$	M1		M1: Two integrals with one in the form $\int f(v)dv$ where $f(v) = v^{\pm\frac{5}{4}}$ or $v^{\pm\frac{4}{5}}$. The other integral must not contain v terms.
	$-\frac{4}{v^{\frac{1}{4}}} = -2t + c$	A1		A1: Correct expression. Condone lack of + c for this A1, but no subsequent marks if no c.
	When $t = 0$, $v = 16 \Rightarrow c = -2$	dM1 A1		dM1: Using $t = 0$ and $v = 16$ to find c. A1: Obtaining $c = -2$.
	$-\frac{4}{v^{\frac{1}{4}}} = -2t - 2$ $v^{\frac{1}{4}} = \frac{2}{1+t}$			
	$v = \left(\frac{2}{t+1}\right)^4$ **AG**	A1	5	A1: Correct final answer. Must see $v^{\frac{1}{4}} = \frac{2}{1+t}$ or $v^{-\frac{1}{4}} = \frac{1+t}{2}$ or $\frac{1}{v^{\frac{1}{4}}} = \frac{1+t}{2}$ Or if they obtain $v = \left(\frac{2}{t+c}\right)^4$ $v = 16, t = 0 \Rightarrow 16^{\frac{1}{4}} = \frac{2}{c}$, condone $c = 1$ (no other root considered)
	Total		**6**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
7(a)	Resolving vertically $T\cos 30 + 20\cos 50 = 4g$	M1A1 A1		M1: Three terms, which must include $4g$, $T\cos\theta$ or $T\sin\theta$ and $20\cos\theta$ or $20\sin\theta$, where θ = 30, 40, 50 or 60. A1: Correct terms A1: Correct equation
	$T\cos 30 = 26.344$ $T = 30.4$ N	A1	4	A1: Correct final answer. Accept 30.4 or AWRT 30.42. Accept 30.4 or 30.5 or AWRT 30.45 from $g = 9.81$.
(b)	Horizontally: $\frac{mv^2}{r} = 20\cos 40 + T\cos 60$	M1 A1F		M1: Three terms, which must include $\frac{mv^2}{r}$ or $\frac{4\times 5^2}{r}$, $T\cos\theta$ or $T\sin\theta$ and $20\cos\theta$ or $20\sin\theta$, where θ = 30, 40, 50 or 60. A1F: Correct equation. May include T, m and v.
	$\frac{4\times 5^2}{r} = 30.53$	dM1		dM1: Substitution of values for T, m and v. Equation of form $\frac{4\times 5^2}{r}$ = number
	$r = 3.27537$ $= 3.28$	A1	4	A1: Correct answer. Accept 3.27 or 3.28 or AWRT 3.28. Accept 3.27 or AWRT 3.27 from $g = 9.81$. **Note: Do not accept $\frac{mv^2}{r} = 30.4$ or similar.**
	Total		**8**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
8(a)	Using conservation of energy (lowest and highest points) $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(2a)$	M1A1		M1: Equation for conservation of energy with two KE terms and one or two PE terms. May see m or 0.3. A1: Correct equation.
	$u^2 = v^2 + 4ag$ For complete revolutions, $v > 0$ $\therefore u^2 > 4ag$ $u > 2\sqrt{ag}$ **AG**	A1	3	A1: Correct result with statement of $v > 0$ and some intermediate working including $4ag$ term.
	Or Use of PE at top and KE at B Correct PE and KE Correct deduction including inequality	(M1) (A1) (A1)		
(b)(i)	C of Energy $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mga(1 + \sin\theta)$	M1A1		M1: Equation for conservation of energy with two KE terms and one or two PE terms including a $\sin\theta$. May see m or 0.3. A1: Correct equation.
	$v^2 = \left(\sqrt{\frac{9}{2}ag}\right)^2 - 2ga(1 + \sin\theta)$ $= \frac{5}{2}ag - 2ag\sin\theta$			
	Resolve radially $\pm R = -mg\sin\theta + \frac{mv^2}{a}$	M1A1		M1: Three term equation from resolving radially. Correct three terms, but condone signs and replacement of sin by cos. A1: Correct equation. May see m or 0.3.
	$= -mg\sin\theta + \frac{5}{2}mg - 2mg\sin\theta$ $= -3mg\sin\theta + \frac{5}{2}mg$			
	$= \left(\frac{3}{4} - \frac{9}{10}\sin\theta\right)g$ **OE** (must include g)	A1	5	A1: Simplified correct final answer. Condone $\left(\frac{9}{10}\sin\theta - \frac{3}{4}\right)g$
(ii)	When this reaction is zero, $\left(\frac{3}{4} - \frac{9}{10}\sin\theta\right)g = 0$	M1		M1: Putting their reaction equal to zero.
	$\sin\theta = \frac{5}{6}$ θ is 56.4° above horizontal	A1	2	A1: Correct angle. Accept AWRT 56.44.
	Total		**10**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
9(a)	EPE $= \frac{\lambda x^2}{2l}$			
	$= \frac{1800\times(4)^2}{2\times 6}$	B1 M1		B1: Extension = 4. M1: Substitution of 6, 1800 and their extension into EPE formula.
	$= 2400$ J	A1	3	A1: Correct EPE
(b)				
	$\frac{1800\times(x)^2}{2\times 6} = \frac{1}{2}\times 200\times 8^2$	M1		M1: Equation with EPE and KE terms, both correct.
	$x^2 = 42.67$ $x = 6.53$ m	A1		A1: Correct extension. Accept $\frac{8\sqrt{6}}{3}$ or 6.53 or AWRT 6.532.
	Distance from O is 12.5 m	A1	3	A1: Correct distance. Accept 12.5 or AWRT 12.53.
(c)	Resistance force is 800 N Work done by resistance force is $800 \times (x + 6)$	B1		B1: Correct work done by resistance force.
	C of Energy gives $\frac{1800\times(x)^2}{2\times 6} + 800\times(x+6) = \frac{1}{2}\times 200\times 8^2$	M1A1 A1		M1: Three energy terms, KE, Work Done and EPE. A1: EPE correct. A1: Correct equation.
	$150x^2 + 800(x+6) = 6400$ $3x^2 + 16x - 32 = 0$ or $150x^2 + 800x - 1600 = 0$	A1		A1: Correct quadratic equation with no brackets.
	$x = \frac{-16 \pm \sqrt{16^2 + 4\times 3\times 32}}{2\times 3}$	dM1		dM1: Solving their quadratic equation with correct formula and correct substitution
	$x = 1.5497$	A1		A1: Correct positive solution stated. Accept 1.54 or 1.55 or AWRT 1.55.
	Distance from O is 7.55 m	A1	8	A1: Correct distance from O. Accept 7.55 or 7.54 or AWRT 7.55.
	OR Use d for distance: $800 \times d$	(B1)		B1: Correct work done by resistance force.
	C of Energy gives $\frac{1800\times(d-6)^2}{2\times 6} + 800\times d = \frac{1}{2}\times 200\times 8^2$	(M1A1) (A1A1)		M1: Three energy terms, KE, Work Done and EPE.
	$150d^2 - 1000d - 1000 = 0$ $3d^2 - 20d - 20 = 0$	(A1)		A1: Seeing $d - 6$ in EPE A1: EPE correct. A1: Correct equation.
	$x = \frac{-20 \pm \sqrt{20^2 + 4\times 3\times 20}}{2\times 3}$	(dM1)		A1: Correct quadratic equation with no brackets.
	$d = 7.55$	(A1)		dM1: Solving their quadratic equation. A1: Correct distance from O. Accept 7.55 or 7.54 or AWRT 7.55.
	Total		**14**	
	TOTAL		**75**	